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14. ABSTRACT While it is in some sense natural that quantum systems are efficient at simulating one another, a less natural question is the efficient simulation of classical systems on quantum computers (QCs). This question was first raised, and partly answered by the PI in the context of Ising spin glasses (an ensemble of classical spin-1/2's with fixed random interactions). Recently, there has been further interest in classical physics simulations on QCs in the context of hydrodynamics, chaos, and knot theory, which has a deep connection to classical statistical mechanics.					
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a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 213-740-0198

## Report Title

Final Report on “Quantum Computational Complexity of Spin Glasses”

### ABSTRACT

While it is in some sense natural that quantum systems are efficient at simulating one another, a less natural question is the efficient simulation of classical systems on quantum computers (QCs). This question was first raised, and partly answered by the PI in the context of Ising spin glasses (an ensemble of classical spin-1/2's with fixed random interactions). Recently, there has been further interest in classical physics simulations on QCs in the context of hydrodynamics, chaos, and knot theory, which has a deep connection to classical statistical mechanics. The purpose of this project is to quantify the computational complexity of the canonical problem of classical statistical mechanics: computation of the classical partition function. We have approached this problem using the Potts model and Ising spin glasses, which are known to give rise to a rich class of hard computational problems. Indeed, instances of spin glass and Potts model problems have been shown to be NP-hard, and have been mapped to problems in graph and knot theory. An “instance” of the spin glass problem refers here to a particular choice of (i) graph describing the spin glass (spins with  $q=2$  states are located on the vertices, interactions on the edges), and (ii) distribution of interactions. An instance of the Potts model refers to a choice of a graph on whose vertices reside spins with  $q>1$  ( $q$  an integer) states. In certain limits of particularly simple graphs and distributions these problems are analytically solvable, while in other limits they are computationally hard; hence by “tuning” the graph and distribution one may expect to traverse a “landscape of hardness”, whose quantum computational complexity we are exploring.

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**List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:**

#### (a) Papers published in peer-reviewed journals (N/A for none)

1. J. Geraci and D.A. Lidar, Comm. Math. Phys. 279, 735-768 (2008)
2. J. Geraci and D.A. Lidar, New J. Physics 12, 075026 (2010)
3. J. Geraci, Quantum Info. Process. 7, 227 (2008)

**Number of Papers published in peer-reviewed journals:** 3.00

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#### (b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

1. J. Geraci and Frank Van Bussel, eprint arXiv:cs/0703129v4

**Number of Papers published in non peer-reviewed journals:** 1.00

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#### (c) Presentations

1. D.A. Lidar, Three quantum lunch problems, Los Alamos “Quantum Lunch” seminar, July 27, 2006.
2. J. Geraci, Weight Enumerators and Statistical Physics via Quantum Computation, Workshop on Quantum Methods for Classical Physics Problems and Differential Equations, Los Alamos, May 25, 2007.

**Number of Presentations:** 2.00

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#### Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

**Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):** 0

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#### Peer-Reviewed Conference Proceeding publications (other than abstracts):

**Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):** 0

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#### (d) Manuscripts

**Number of Manuscripts:**

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## Patents Submitted

## Patents Awarded

## Awards

## Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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**FTE Equivalent:**

**Total Number:**

## Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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**FTE Equivalent:**

**Total Number:**

## Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
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Daniel Lidar

No

**FTE Equivalent:**

**Total Number:**

1

## Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
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**FTE Equivalent:**

**Total Number:**

### Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: .....

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): .....

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: .....

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense .....

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: .....

### Names of Personnel receiving masters degrees

NAME

**Total Number:**

### Names of personnel receiving PhDs

NAME

Joseph Geraci

**Total Number:**

1

### Names of other research staff

NAME

PERCENT SUPPORTED

**FTE Equivalent:**

**Total Number:**

### Sub Contractors (DD882)

### Inventions (DD882)

## Scientific Progress

### Accomplishments:

- We identified a class of graphs for which there is a quantum speedup relative to classical computers. The paper describing this work was published in Comm. Math. Phys. 279, 735-768 (2008) [also quant-ph/0703023]. In this work we presented an efficient quantum algorithm for the exact evaluation of either the fully ferromagnetic or anti-ferromagnetic q-state Potts partition function  $Z$  for a family of graphs related to irreducible cyclic codes. This problem is related to the evaluation of the Jones and Tutte polynomials. We considered the connection between the weight enumerator polynomial from coding theory and  $Z$  and exploited the fact that there exists a quantum algorithm for efficiently estimating Gauss sums in order to obtain the weight enumerator for a certain class of linear codes. In this way we demonstrated that for a certain class of sparse graphs, which we called Irreducible Cyclic Cocycle Code (ICCC\_ $\epsilon$ ) graphs, quantum computers provide a polynomial speed up in the difference between the number of edges and vertices of the graph, and an exponential speed up in  $q$ , over the best classical algorithms known to date.
- We used the classical Ising model to determine limits to the power of quantum computation. This work was published in New J. Physics 12, 075026 (2010) [also arXiv:0902.4889]. In this work we exploited a recently constructed mapping between quantum circuits and graphs in order to prove that circuits corresponding to certain planar graphs can be efficiently simulated classically. The proof uses an expression for the Ising model partition function in terms of quadratically signed weight enumerators (QWGTs), which are polynomials that arise naturally in an expansion of quantum circuits in terms of rotations involving Pauli matrices. We combined this expression with a known efficient classical algorithm for the Ising partition function of any planar graph in the absence of an external magnetic field, and the Robertson-Seymour theorem from graph theory. We gave as an example a set of quantum circuits with a small number of non-nearest neighbor gates which admit an efficient classical simulation.
- We proved a theorem on the quantum evaluation of weight enumerators for a certain class of Cyclic Codes. This work is available as eprint arXiv:cs/0703129v4. This note is a stripped down version of our published paper on the Potts partition function, where we concentrated solely on the linear coding aspect of our approach. It is meant as a resource for people interested in coding theory but who do not know much of the mathematics involved and how quantum computation may provide a speed up in the computation of a very important quantity in coding theory. We provided a theorem on the quantum computation of the Weight Enumerator polynomial for a restricted family of cyclic codes. The complexity of obtaining an exact evaluation is  $O(k^{2s}(\log q)^2)$ , where  $s$  is a parameter which determines the class of cyclic codes in question,  $q$  is the characteristic of the finite field over which the code is defined, and  $k$  is the dimension of the code. We also provided an overview of cyclotomic cosets and discussed applications including how they can be used to speed up the computation of the weight enumerator polynomial (which is related to the Potts partition function). We also gave an algorithm which returns the coset leaders and the size of each coset from the list  $\{0, 1, 2, \dots, N-1\}$ , whose time complexity is  $\text{soft-}O(N)$ . This algorithm uses standard techniques but we included it as a resource for students. Note that cyclotomic cosets do not improve the asymptotic complexity of the computation of weight enumerators.
- We found a BQP-complete problem related to the Ising model partition function via a new connection between quantum circuits and graphs. This work was published in Quantum Info. Process. 7, 227 (2008) [also arXiv:0801.4833v3]. In this work we presented a simple construction that maps quantum circuits to graphs and vice-versa. Inspired by the our earlier results linking the Ising partition function with quadratically signed weight enumerators (QWGTs), we also presented a BQP-complete problem for the additive approximation of a function over hypergraphs related to the generating function of Eulerian subgraphs for ordinary graphs. We discussed connections with the Ising partition function.

## Technology Transfer

## Final Report on “Quantum Computational Complexity of Spin Glasses”

**Principal Investigator:** Daniel Lidar, University of Southern California

**Senior Investigators:** None

**Postdoctoral Associates:** None

**Graduate Student:** Joseph Geraci

**Type of Award:** QA

**Start Date:** August 1, 2005

**End Date:** July 31, 2008

### Main Project Goals:

Find and analyze quantum algorithms for efficient solution of problems in classical statistical mechanics

### Project Description:

While it is in some sense natural that quantum systems are efficient at simulating one another, a less natural question is the efficient simulation of *classical* systems on quantum computers (QCs). This question was first raised, and partly answered by the PI in the context of Ising spin glasses (an ensemble of classical spin-1/2's with fixed random interactions). Recently, there has been further interest in classical physics simulations on QCs in the context of hydrodynamics, chaos, and knot theory, which has a deep connection to classical statistical mechanics. *The purpose of this project is to quantify the computational complexity of the canonical problem of classical statistical mechanics: computation of the classical partition function.* We have approached this problem using the Potts model and Ising spin glasses, which are known to give rise to a rich class of hard computational problems. Indeed, instances of spin glass and Potts model problems have been shown to be NP-hard, and have been mapped to problems in graph and knot theory. An “instance” of the spin glass problem refers here to a particular choice of (i) graph describing the spin glass (spins with  $q=2$  states are located on the vertices, interactions on the edges), and (ii) distribution of interactions. An instance of the Potts model refers to a choice of a graph on whose vertices reside spins with  $q>1$  ( $q$  an integer) states. In certain limits of particularly simple graphs and distributions these problems are analytically solvable, while in other limits they are computationally hard; hence by “tuning” the graph and distribution one may expect to traverse a “landscape of hardness”, whose quantum computational complexity we are exploring.

### Accomplishments:

- We identified a class of graphs for which there is a quantum speedup relative to classical computers. The paper describing this work was published in Comm. Math. Phys. **279**, 735-768 (2008) [also quant-ph/0703023]. In this work we presented an efficient quantum algorithm for the exact evaluation of either the fully ferromagnetic or anti-ferromagnetic  $q$ -state Potts partition function  $Z$  for a family of graphs related to irreducible cyclic codes. This problem is related to the evaluation of the Jones and Tutte polynomials. We considered the connection between the weight enumerator polynomial from coding theory and  $Z$  and exploited the fact that there exists a quantum algorithm for efficiently estimating Gauss sums in order to obtain the weight enumerator for a certain

class of linear codes. In this way we demonstrated that for a certain class of sparse graphs, which we called Irreducible Cyclic Cocycle Code (ICCC\_ $\epsilon$ ) graphs, quantum computers provide a polynomial speed up in the difference between the number of edges and vertices of the graph, and an exponential speed up in  $q$ , over the best classical algorithms known to date.

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- We proved a theorem on the quantum evaluation of weight enumerators for a certain class of Cyclic Codes. This work is available as eprint arXiv:cs/0703129v4. This note is a stripped down version of our published paper on the Potts partition function, where we concentrated solely on the linear coding aspect of our approach. It is meant as a resource for people interested in coding theory but who do not know much of the mathematics involved and how quantum computation may provide a speed up in the computation of a very important quantity in coding theory. We provided a theorem on the quantum computation of the Weight Enumerator polynomial for a restricted family of cyclic codes. The complexity of obtaining an exact evaluation is  $O(k^{2s}(\log q)^2)$ , where  $s$  is a parameter which determines the class of cyclic codes in question,  $q$  is the characteristic of the finite field over which the code is defined, and  $k$  is the dimension of the code. We also provided an overview of cyclotomic cosets and discussed applications including how they can be used to speed up the computation of the weight enumerator polynomial (which is related to the Potts partition function). We also gave an algorithm which returns the coset leaders and the size of each coset from the list  $\{0,1,2,\dots,N-1\}$ , whose time complexity is  $\text{soft-}O(N)$ . This algorithm uses standard techniques but we included it as a resource for students. Note that cyclotomic cosets do not improve the asymptotic complexity of the computation of weight enumerators.

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function of Eulerian subgraphs for ordinary graphs. We discussed connections with the Ising partition function.

## **Major Project Milestones**

Year One (completed):

- Showed that spin glass partition function is special case of quadratically signed weight enumerators:
- Modulus of Potts partition function for any planar graph may be approximated efficiently by a QC at roots of unity:
- Proved a theorem establishing a quantum computational complexity classification of classical Potts models by graph type

Year Two (completed):

- “Investigate in detail the computational complexity classification we have obtained for Potts models on graphs. Identify in particular which graphs are classically hard”: Accomplished. We now know that the class ICCC we have defined is subject to a quantum speedup (polynomial in the number of vertices and exponential in the Potts spin variable).
- “Explore the relationship between the Ising spin-glass problem and quadratically signed weight enumerators (QWGTs). Find specific examples of spin glass classes in BQP. Establish a connection between quantum error correction codes and spin glasses (via weight enumerators of codes) and attempt a classification of spin glasses in terms of the Gottesman-Knill theorem”: Partly accomplished. We now know that the QWGT approach is unlikely to yield examples of a quantum speedup. On the other hand we can use the connection between QWGTs and Ising spin glasses to classify quantum circuits with an efficient classical simulation, using a route that is totally distinct from the Gottesman-Knill theorem.

Year Three (completed):

- Determined instances of the Potts and Ising model for which QCs outperform classical machines in the evaluation of partition functions. Did this specifically for graphs in ICCC.
- Found a new criterion for determining when a quantum circuit can be classically simulated, by a mapping to easily computable partition functions. Did this using the connection between QWGTs and the Ising spin glass partition function.
- Extended our approach to knot theoretical considerations and the Jones polynomial.

## **Presentations**

- D.A. Lidar, *Three quantum lunch problems*, Los Alamos “Quantum Lunch” seminar, July 27, 2006.
- J. Geraci, *Weight Enumerators and Statistical Physics via Quantum Computation*, Workshop on Quantum Methods for Classical Physics Problems and Differential Equations, Los Alamos, May 25, 2007.